Finding the Domain of a Function

The domain is the set of input values for the function. When we are asked to find the domain of a function, we want to find the values of x (the independent variable) that are "legal" to input into the function.

Example 0: $f(x) = x^2 + 3x - 2$ Domain : All real numbers $(-\infty, \infty)$

Example 1: $f(x) = \frac{1}{x}$ Domain :{x such that $x \neq 0$} $(-\infty, 0) \cup (0, \infty)$

Example 2: $f(x) = \sqrt{x}$ Domain : {x such that $x \geq 0$} $[0, \infty)$

Example 3: $f(x) = \sqrt{x-2}$ Domain : {x such that $x \geq 2$} $[2, \infty)$

Example 4: $f(x) = \sqrt{x+3}$ Domain : {x such that $x \geq -3$} $[-3, \infty)$

Example 5: $f(x) = \sqrt{12-x}$ Domain : {x such that $x \leq 12$} $(-\infty, 12]$

Example 6: $f(x) = \sqrt{20+x}$ Domain : {x such that $x \geq -20$} $[20, \infty)$

Sometimes a function has more than one restriction that affects its domain

Example 7: $f(x) = \frac{1}{\sqrt{x+3}}$ Domain : {x such that $x > -3$} $(-3, \infty)$

Example 8: $f(x) = \frac{\sqrt{x-2}}{x-4}$ Domain : {x such that $x \geq 2$ and $x \neq 4$} $[2, 4) \cup (4, \infty)$

Example 9: $f(x) = \frac{\sqrt{x-2}}{x+1}$ Domain : {x such that $x \geq 2$ and $x \neq -1$} which is {x such that $x \geq 2$ } $[2, \infty)$

Example 10: $f(x) = \frac{\sqrt[4]{x-2}}{x^2-3x-4} = \frac{\sqrt[4]{x-2}}{(x+1)(x-4)}$ Domain : {x such that $x \geq 2$ and $x \neq 4$} $[2, 4) \cup (4, \infty)$

Example 11: $f(x) = \frac{\sqrt[3]{x-2}}{x^2-3x-4} = \frac{\sqrt[3]{x-2}}{(x+1)(x-4)}$ Domain : {x such that $x \neq -1$ and $x \neq 4$}

$(-\infty, -1) \cup (-1, 4) \cup (4, \infty)$

Example 12: $f(x) = \log x$ Domain : {x such that $x > 0$ } $(0, \infty)$

Check for even roots: the radicand (or base if in exponential form) can not be negative for even roots, but can be negative for odd roots.

(Even roots must have input ≥ 0; odd roots can have input <0 or ≥ 0)

Check for division by 0: Set the denominator = 0 and solve for x. EXCLUDE those values from the domain.

Check for log functions: The input into a log function must be POSITIVE

Finding the Range of a Function

The range is the set of output values for the function. When we are asked to find the range of a function, we want to find all the values of the dependent (output) variable of that can be obtained as output from the function.

Example 1: $y = f(x) = 2x$ Range : All real numbers $(-\infty, \infty)$

Example 2: $y = f(x) = x^2$ Range : {y such that $y \geq 0$} $[0, \infty)$

Example 3: $y = f(x) = x^2 + 2$ Range : {y such that $y \geq 2$} $[2, \infty)$

Example 4: $y = f(x) = -3$ Range : $\{-3\}$

Example 5: $y = f(x) = \sqrt{x}$ Range : {y such that $y \geq 0$} $[0, \infty)$

Example 6: $y = f(x) = \sin x$ Range : {y such that $-1 \leq y \leq 1$} $[-1, 1]$