Chapter 9 Section 4
Properties of Logarithms

Product Rule
Properties of exponents correspond to properties of logarithms.

Example: $b^m \bullet b^n = b^{m+n}$
Bases are the same, add the exponents.

The Product Rule:
Let b, M, and N be positive real numbers with b ≠ 1

$$\log_b(MN) = \log_b M + \log_b N$$

The logarithm of a product is the sum of the logarithms.

When write a single logarithm as two or more logarithms it is said that one is expanding a logarithmic expression.

Expand each logarithmic expression
a) $\log_4(7 \bullet 5)$ b) log (10x)

The Quotient Rule

Divide exponential expression with the same base, subtract exponents.

$$\frac{b^m}{b^n} = b^{m-n}$$

The Quotient Rule
Let b, M, and N be positive real numbers with b ≠ 1

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

The logarithm of a quotient is the difference of the logarithm.

Example:
Expand the expression

$$\log\frac{x}{19}$$

Solution:

Log x – log 19

Expand each logarithmic expression:

a) $\log_7\left(\frac{19}{x}\right)$

b) $\ln\left(\frac{e^3}{7}\right)$

The Power Rule:
Exponential expression is raised to a power, multiply exponents

$$\left(b^m\right)^n = b^{mn}$$

The Power Rule:
Let b and M be positive real numbers with b ≠ 1, and let p be any read number,

$$\log_b M^p = p\ \log_b M$$

The logarithm of a number with an exponent is the product of the exponent and the logarithm of that number.

Pull the exponent to the front.

Example:

$$\ln\ x^2 = 2\ \ln x$$

Try:

a) $\log_5 7^4$

b) $\ln\sqrt{x}$

c) $\log\left(4x\right)^5$

Expanding Logarithmic Expressions

Properties for Expanding Logarithmic Expressions

For $M > 0$ and $N > 0$:

1. $\log_b(MN) = \log_b M + \log_b N$ — Product rule
2. $\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$ — Quotient rule
3. $\log_b M^p = p \log_b M$ — Power rule

Expand each expression as much as possible

a) $\log_b\left(x^2\sqrt{y}\right)$ b) $\log_6\left(\frac{\sqrt[3]{x}}{36y^4}\right)$

Condensing Logarithmic Expressions

To condense a logarithmic express, write the expression as a single logarithmic expression.

Properties for Condensing Logarithmic Expressions

For $M > 0$ and $N > 0$:

1. $\log_b M + \log_b N = \log_b(MN)$ Product rule
2. $\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$ Quotient rule
3. $p \log_b M = \log_b M^p$ Power rule

Write as a single logarithm

a) $\log_4 2 + \log_4 32$ b) log (4x – 3) – log x

Coefficients of logarithms must be 1 before you can condense them using the product and quotient rule.

Example:

2 ln x + ln (x + 1)

$\ln x^2 + \ln(x+1)$

$\ln\left[x^2(x+1)\right]$

Write as a single logarithm

a) $\frac{1}{2}\log x + 4\log(x-1)$ b) 3 ln (x + 7) – ln x c) $4\log_b x - 2\log_b 6 - \frac{1}{2}\log_b y$

The Change-of-Base Property
Calculators can give the values of both common logarithms (base 10) and natural logarithms (base e). To find the logarithm of any other base, use the change-of-base property.

The Change-of-Base Property
For any logarithmic base 'a' and 'b', and any positive number M,

$$\log_b M = \frac{\log_a M}{\log_a b}$$

Base 'b' is the base of the original logarithm. Base 'a' is a new base that is introduced. This property allows to change from base 'b' to any new base 'a' as long as the newly introduced base is a positive and not equal to 1.

This property is used to write a logarithm in terms of quantities that can be evaluated with a calculator.

The Change-of-Base Property: Introducing Common and Natural Logarithms

Introducing Common Logarithms

$$\log_b M = \frac{\log M}{\log b}$$

Introducing Natural Logarithms

$$\log_b M = \frac{\ln M}{\ln b}$$

Use common logarithms to evaluate $\log_5 140$

Solution:
Use the change-of-base property to change to the common logarithm or natural logarithm.

$$\log_5 140$$

$$\frac{\log 140}{\log 5}$$

Use your calculator to find the values of log 140 and log 5, then divide.

Evaluate to four decimal places.
a) $\log_{14} 87.5$ b) $\log_{0.3} 19$