Chapter 9 section 2
Composite and Inverse Functions.

Composite function

Notation: f(g(x)), read the composition of the function f with g.
f of g of x

$(f \circ g)(x) = f(g(x))$

Read f of g of x OR f composed with g at x.

Evaluate:

$(f \circ g)(x) = 0.85x - 300$

if x = 1400

Solution:

$(f \circ g)(1400) = 0.85(1400) - 300$

The Composition of Functions

The **composition of the function *f* with *g*** is denoted by $f \circ g$ and is defined by the equation

$$(f \circ g)(x) = f(g(x)).$$

The **domain of the composite function $f \circ g$** is the set of all x such that

1. x is in the domain of g and
2. $g(x)$ is in the domain of f.

Forming Composite Functions

Given: f(x) = 3x – 4 and g(x) = $x^2 + 6$

Find: $(f \circ g)(x)$

Solution

$(f \circ g)(x)$ means f(g(x))

f(x) = 3x – 4, so replace x with g(x)
f(g(x) = 3(g(x) – 4 and g(x) = x^2+6

so $(f \circ g)(x)$ = f(g(x) = 3(g(x)) – 4

$3(x^2+6) - 4$
$3x^2 + 18 - 4$
$3x^2 + 14$

Try: Given g(x) = 5x + 6 and g(x) = x^2 - 1 find
a) $(f \circ g)(x)$ b) $(g \circ f)(x)$

Inverse functions

Definition of the Inverse of a Function

Let f and g be two functions such that

$$f(g(x)) = x \qquad \text{for every } x \text{ in the domain of } g$$

and

$$g(f(x)) = x \qquad \text{for every } x \text{ in the domain of } f.$$

The function g is the **inverse of the function *f***, and is denoted by f^{-1} (read "f-inverse"). Thus, $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. The domain of f is equal to the range of f^{-1}, and vice versa.

Show that f and g are inverses of each other, show
f(g(x)) = x and g(f(x)) = x

Example
Show that f(x) = 5x and g(x) = $\frac{x}{5}$ are inverses.

Must show f(g(x)) = x and g(f(x)) = x

Finding the Inverse of a Function.

Finding the Inverse of a Function

The equation for the inverse of a function f can be found as follows:

1. Replace $f(x)$ with y in the equation for $f(x)$.
2. Interchange x and y.
3. Solve for y. If this equation does not define y as a function of x, the function f does not have an inverse function and this procedure ends. If this equation does define y as a function of x, the function f has an inverse function.
4. If f has an inverse function, replace y in step 3 with $f^{-1}(x)$. We can verify our result by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

Example: Find the inverse of $f(x) = 7x - 5$

Solution:
1) Replace f(x) with y: $y = 7x - 5$

2) Interchange x and y: $x = 7y - 5$

3) Solve for y: $y = \frac{x+5}{7}$

4) Replace y with $f^{-1}(x)$: $f^{-1}(x) = \frac{x+5}{7}$

Try:
Find the inverse of $f(x) = 2x + 7$

The Horizontal Line Test and One-to-One Functions

Horizontal Line Test is used to determine if a function has an inverse.

Horizontal line intersects the graph at no more than one point.

Function f has an inverse that is a function, f^{-1} , if there is no horizontal line that intersects the graph of the function f at more than one point.

Any function that passes the horizontal line test is a one-to-one function.

Graph the Inverse Function

Make a table of values of the function and the inverse, then graph.