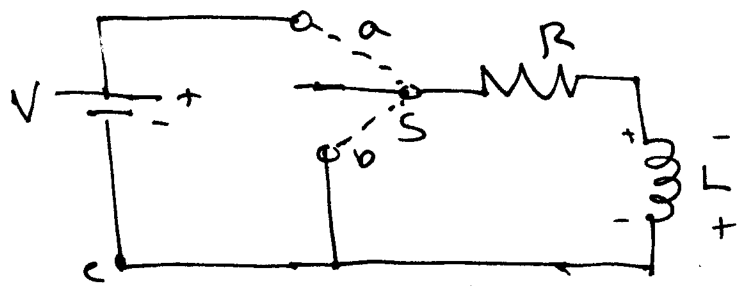
RL Circuits



$$\mathcal{E}_L = -L\frac{dI}{dt}$$

Position "a"

$$\Sigma V_{loop} = 0$$

$$V - IR - L\frac{dI}{dt} = 0$$

$$\left[\frac{dI}{dt} = \frac{V}{L} - \frac{IR}{L}\right]$$

At $t=0$ when S is closed $I=0$:

$$\boxed{\frac{dI}{dt} = \frac{V}{L}} \quad \text{At } t=0$$

- The larger the inductance L the more opposition to the increase in current and thus the more slowly the current increases.

- As I increases, $\frac{dI}{dt} \to 0$ and the current reaches its steady state value:

$$0 = \frac{V}{L} - \frac{IR}{L}$$

$$\boxed{I = \frac{V}{R}} \quad \text{steady-state current}$$

$$\frac{dI}{dt} = \frac{VR}{LR} - \frac{IR}{L} = -\frac{R}{L}\left(I - \frac{V}{R}\right)$$

$$\int_0^I \frac{dI}{I - \frac{V}{R}} = -\frac{R}{L}\int_0^t dt$$

$$\ln\left(I - \frac{V}{R}\right)\Big|_0^I = -\frac{R}{L}t$$

$$\ln\left(I - \frac{V}{R}\right) - \ln\left(-\frac{V}{R}\right) = -t/\tau$$

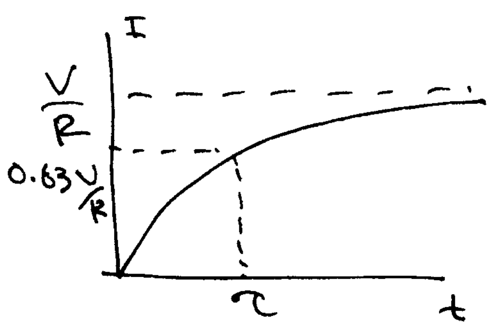
where $\tau = \frac{L}{R}$ (time constant)

$$\ln\left(-\frac{IR}{V} + 1\right) = -t/\tau$$

$$1 - \frac{IR}{V} = e^{-t/\tau}$$

$$\frac{IR}{V} = \left(1 - e^{-t/\tau}\right)$$

$$\boxed{I(t) = \frac{V}{R}\left(1 - e^{-t/\tau}\right)}$$



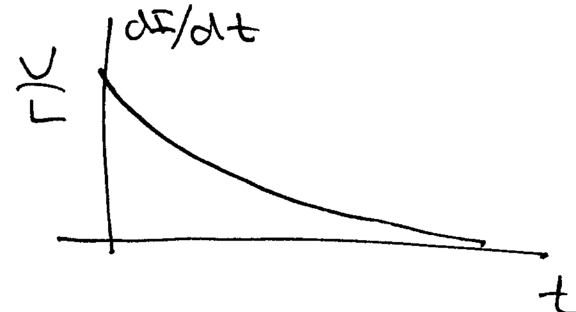
$$I(\tau) = \frac{V}{R}\left(1 - e^{-1}\right)$$

$$I(\tau) = 0.63\frac{V}{R}$$

$$\frac{dI}{dt} = -\frac{V}{R}\left(-\frac{1}{\tau}\right)e^{-t/\tau}$$

$$= \frac{V}{\cancel{R}}\left(\frac{\cancel{R}}{L}\right)e^{-t/\tau}$$

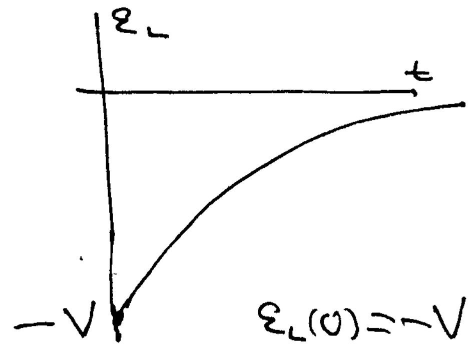
$$\boxed{\frac{dI}{dt} = \frac{V}{L}e^{-t/\tau}}$$



$$\varepsilon_L = -L\frac{dI}{dt}$$

$$\varepsilon_L = -L\left(\frac{V}{L}\right)e^{-t/\tau}$$

$$\boxed{\varepsilon_L = -V e^{-t/\tau}}$$



$\varepsilon_L(0) = -V$

Position "b"

$$-IR - L\frac{dI}{dt} = 0$$

$$L\frac{dI}{dt} = -IR$$

$$\int_{V/R}^{I} \frac{dI}{I} = -\frac{R}{L}\int_{0}^{t} dt$$

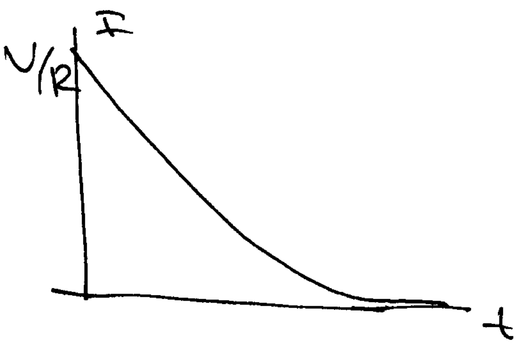
$$\ln I \Big|_{V/R}^{I} = -t/\tau$$

$$\ln I - \ln V/R = -t/\tau$$

$$\ln\left(\frac{IR}{V}\right) = -t/\tau$$

$$I\frac{R}{V} = e^{-t/\tau}$$

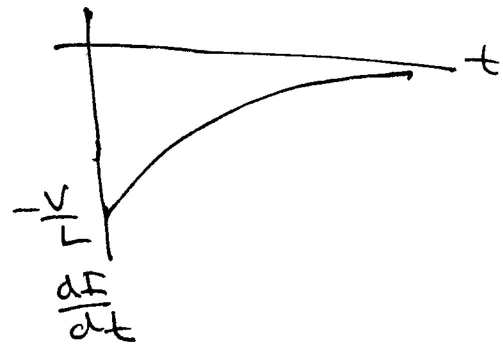
$$\boxed{I(t) = \frac{V}{R}e^{-t/\tau}}$$



$$\frac{dI}{dt} = -\frac{V}{R}\left(\frac{1}{\tau}\right)e^{-t/\tau}$$

$$= -\frac{V}{\cancel{R}}\frac{\cancel{R}}{L}e^{-t/\tau}$$

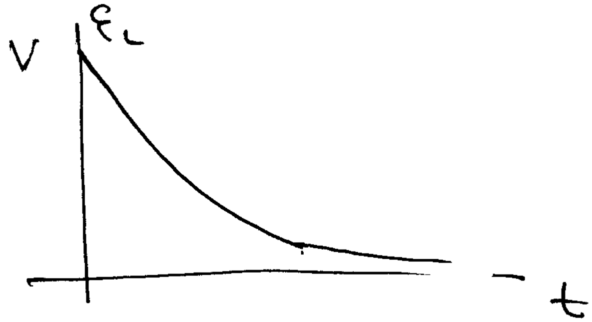
$$\left[\frac{dI}{dt} = -\frac{V}{L}e^{-t/\tau}\right]$$



$$\mathcal{E}_L = -L\frac{dI}{dt}$$

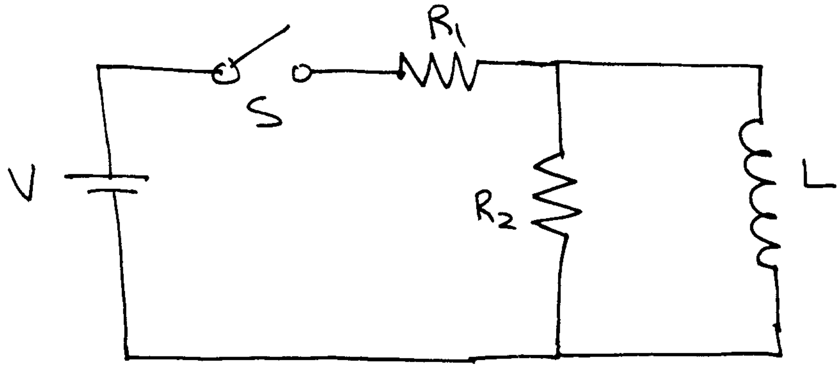
$$\mathcal{E}_L = -L\left(-\frac{V}{L}\right)e^{-t/\tau}$$

$$\left[\mathcal{E}_L = Ve^{-t/\tau}\right]$$



$$\mathcal{E}_L = -L\left(\frac{-IR}{\tau}\right)$$

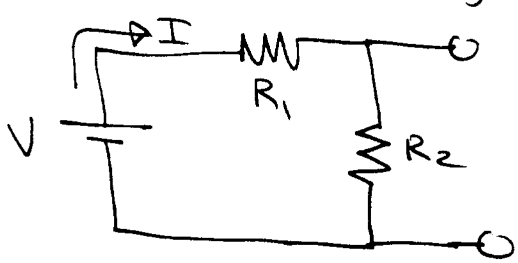
Ex.



$$\boxed{\mathcal{E}_L = -L\frac{dI}{dt}}$$

a) Calculate current through each element just after S is closed.

Since "I" cannot change inst. thru an inductor, the "I" = 0 thru L just after S is closed.



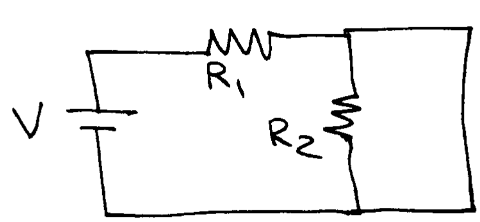
Inductor acts like an open circuit.

$$I_{R1} = I_{R2} = \frac{V}{R_1 + R_2}$$

$$I_L = 0$$

b) Calculate current for $t \gg 0$ after closed.

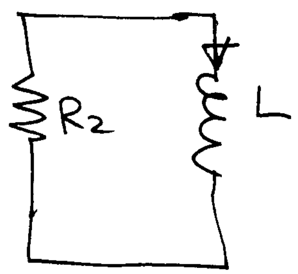
$$\frac{dI}{dt} = 0 \implies \mathcal{E}_L = 0$$ $\Big\{$ Inductor acts like a wire



$$I_{R2} = 0$$

$$I_{R1} = I_L = \frac{V}{R_1}$$

Long after S is closed, it is opened again:
c) Find current just after opened.



$$I_L = \frac{V}{R_1} = I_{R2}$$

$$V_{R2} = I_{R2} R_2$$

$$V_{R2} = \left(\frac{V}{R_1}\right) R_2$$

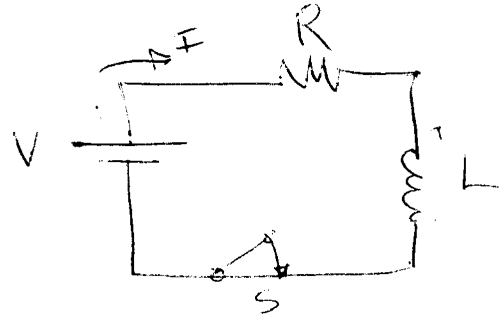
If there is no resistor connected across inductor $R_2 = \infty$ and $V \to \infty$ (very large).
(This can be very dangerous)

Magnetic Energy

- Because the emf induced in an inductor prevents a battery from establishing an instantaneous current, the battery must do work against the inductor to create a current.

- Part of the energy supplied by the battery appears as internal energy in the resistor, while the remaining energy is stored in the B-field.

Recall:



$$V = IR + L\frac{dI}{dt}$$

$$IV = I^2R + LI\,dI/dt$$

This equation tells us that the rate at which energy is supplied by the battery equals the sum of the rate at which energy is delivered to the resistor and the rate at which energy is stored in the inductor

If "u" is the energy stored in the inductor at any time, then:

$$\frac{du_L}{dt} = L I \frac{dI}{dt}$$

$$du = L I \, dI$$

$$u_L = \int_0^I L I \, dI = \frac{1}{2} L I^2$$

$$\boxed{u = \frac{1}{2} L I^2}$$ Energy stored in an inductor

We can also determine the energy density of a magnetic field. For simplicity let's consider a solenoid.

$$L = \mu_0 n^2 A l$$

$$B = \mu_0 I n \implies I = \frac{B}{\mu_0 n}$$

$$u = \frac{1}{2} L I^2 = \frac{1}{2} \mu_0 n^2 A l \left(\frac{B}{\mu_0 n}\right)^2 = \frac{1}{2} \frac{A l B^2}{\mu_0}$$

$$V = A l$$

$$u_B = \frac{u}{V} = \frac{B^2}{2\mu_0}$$

$$\boxed{u_B = \frac{B^2}{2\mu_0}}$$ Magnetic Energy density