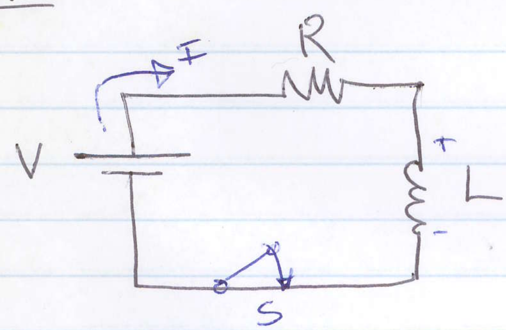
Magnetic Energy

- Because the emf induced in an inductor prevents a battery from establishing an instantaneous current, the battery must do work against the inductor to create a current.

- Part of the energy supplied by the battery appears as internal energy in the resistor, while the remaining energy is stored in the B-field.

Recall:



$$V = IR + L\frac{dI}{dt}$$

$$IV = I^2R + LI\, dI/dt$$

This equation tells us that the rate at which energy is supplied by the battery equals the sum of the rate at which energy is delivered to the resistor and the rate at which energy is stored in the inductor.

If "u" is the energy stored in the inductor at any time, then:

$$\frac{du_L}{dt} = LI\frac{dI}{dt}$$

$$du_L = LI\,dI$$

$$u_L = \int_0^I LI\,dI = \frac{1}{2}LI^2$$

$$\boxed{u = \frac{1}{2}LI^2}$$ Energy stored in an inductor

We can also determine the energy density of a magnetic field. For simplicity let's consider a solenoid.

$L = \mu_0 \frac{N^2 A}{l}$

$$L = \mu_0 n^2 A l$$

$$B = \mu_0 I n \Rightarrow I = \frac{B}{\mu_0 n}$$

$L = \mu_0 n^2 A l$

$$u = \frac{1}{2}LI^2 = \frac{1}{2}\mu_0 n^2 A l\left(\frac{B}{\mu_0 n}\right)^2 = \frac{1}{2}\frac{AlB^2}{\mu_0}$$

$$V = Al$$

$u_E = \frac{1}{2}\epsilon_0 E^2$

$$u_B = \frac{u}{V} = \frac{B^2}{2\mu_0}$$

$$\boxed{u_B = \frac{B^2}{2\mu_0}}$$ Magnetic Energy density