AC CIRCUITS

We would like to analyze the voltage-current relations for individual circuit elements carrying an AC-current.

33.2 Resistors in an AC Circuit

Consider a simple AC circuit consisting of a resistor and an AC source as shown in Figure 33.2. At any instant, the algebraic sum of the voltages around a closed loop in a circuit must be zero (Kirchhoff's loop rule). Therefore, $\Delta v + \Delta v_R = 0$ or, using Equation 27.7 for the voltage across the resistor,

$$\Delta v - i_R R = 0$$

If we rearrange this expression and substitute $\Delta V_{max} \sin \omega t$ for Δv, the instantaneous current in the resistor is

$$i_R = \frac{\Delta v}{R} = \frac{\Delta V_{max}}{R} \sin \omega t = I_{max} \sin \omega t \qquad (33.1)$$

where I_{max} is the maximum current:

$$I_{max} = \frac{\Delta V_{max}}{R} \qquad (33.2)$$

◀ Maximum current in a resistor

Equation 33.1 shows that the instantaneous voltage across the resistor is

$$\Delta v_R = i_R R = I_{max} R \sin \omega t \qquad (33.3)$$

◀ Voltage across a resistor

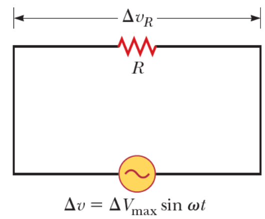
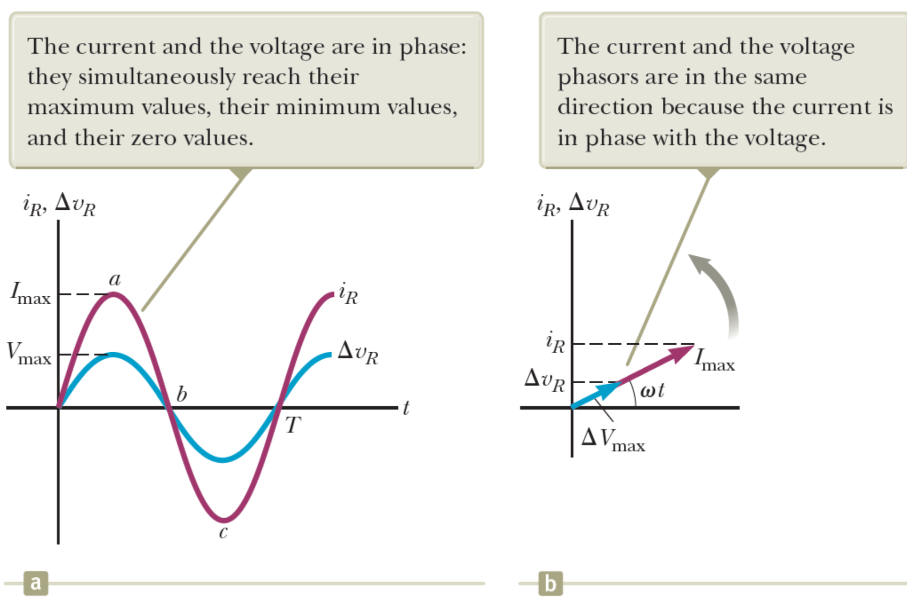


Figure 33.2 A circuit consisting of a resistor of resistance R connected to an AC source, designated by the symbol.



Since I_R and V_R both vary as sin (wt) and they reach their maximum values at the same time, we say that the current and voltage across a resistor are in phase.

Resistors behave the same in AC and DC circuits. However, capacitors and inductors don't!

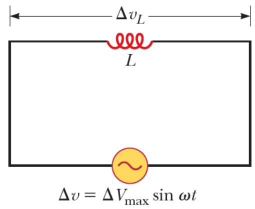


Figure 33.6 A circuit consisting of an inductor of inductance L connected to an AC source.

33.3 Inductors in an AC Circuit

Now consider an AC circuit consisting only of an inductor connected to the terminals of an AC source as shown in Figure 33.6. Because $\Delta v_L = -L(di_L/dt)$ is the self-induced instantaneous voltage across the inductor (see Eq. 32.1), Kirchhoff's loop rule applied to this circuit gives $\Delta v + \Delta v_L = 0$, or

$$\Delta v - L\frac{di_L}{dt} = 0$$

Substituting $\Delta V_{max} \sin \omega t$ for Δv and rearranging gives

$$\Delta v = L\frac{di_L}{dt} = \Delta V_{max} \sin \omega t \qquad \textbf{(33.6)}$$

Solving this equation for di_L gives

$$di_L = \frac{\Delta V_{max}}{L} \sin \omega t \, dt$$

Integrating this expression[1] gives the instantaneous current i_L in the inductor as a function of time:

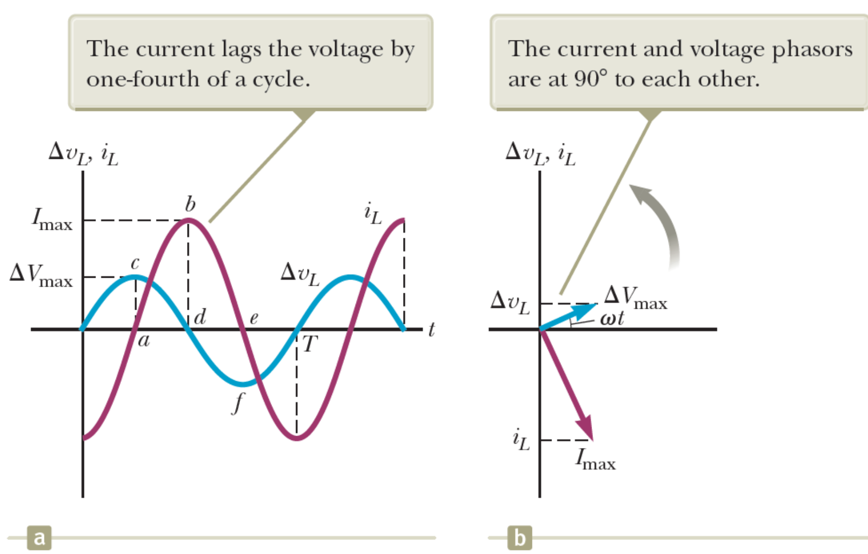
$$i_L = \frac{\Delta V_{max}}{L}\int \sin \omega t \, dt = -\frac{\Delta V_{max}}{\omega L} \cos \omega t \qquad \textbf{(33.7)}$$

Using the trigonometric identity $\cos \omega t = -\sin(\omega t - \pi/2)$, we can express Equation 33.7 as

Current in an inductor ▶

$$i_L = \frac{\Delta V_{max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right) \qquad \textbf{(33.8)}$$

Comparing this result with Equation 33.6 shows that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are *out* of phase by $\pi/2$ rad $= 90°$.



<u>For an AC applied voltage, the current in an inductor always lags behind the voltage across the inductor by 90°.</u>

Equation 33.7 shows that the current in an inductive circuit reaches its maximum value when $\cos \omega t = \pm 1$:

$$I_{\max} = \frac{\Delta V_{\max}}{\omega L} \tag{33.9}$$

$$X_L \equiv \omega L \tag{33.10}$$

◀ **Inductive reactance**

Therefore, we can write Equation 33.9 as

$$I_{\max} = \frac{\Delta V_{\max}}{X_L} \tag{33.11}$$

Using Equations 33.6 and 33.11, we find that the instantaneous voltage across the inductor is

Voltage across an inductor ▶

$$\Delta v_L = -L\frac{di_L}{dt} = -\Delta V_{\max} \sin \omega t = -I_{\max} X_L \sin \omega t \tag{33.12}$$

33.4 Capacitors in an AC Circuit

Figure 33.9 shows an AC circuit consisting of a capacitor connected across the terminals of an AC source. Kirchhoff's loop rule applied to this circuit gives $\Delta v + \Delta v_C = 0$, or

$$\Delta v - \frac{q}{C} = 0 \tag{33.13}$$

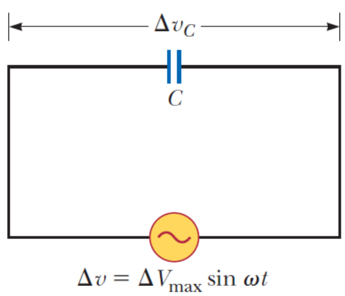


Figure 33.9 A circuit consisting of a capacitor of capacitance C connected to an AC source.

Substituting $\Delta V_{max} \sin \omega t$ for Δv and rearranging gives

$$q = C \Delta V_{max} \sin \omega t \tag{33.14}$$

where q is the instantaneous charge on the capacitor. Differentiating Equation 33.14 with respect to time gives the instantaneous current in the circuit:

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{max} \cos \omega t \tag{33.15}$$

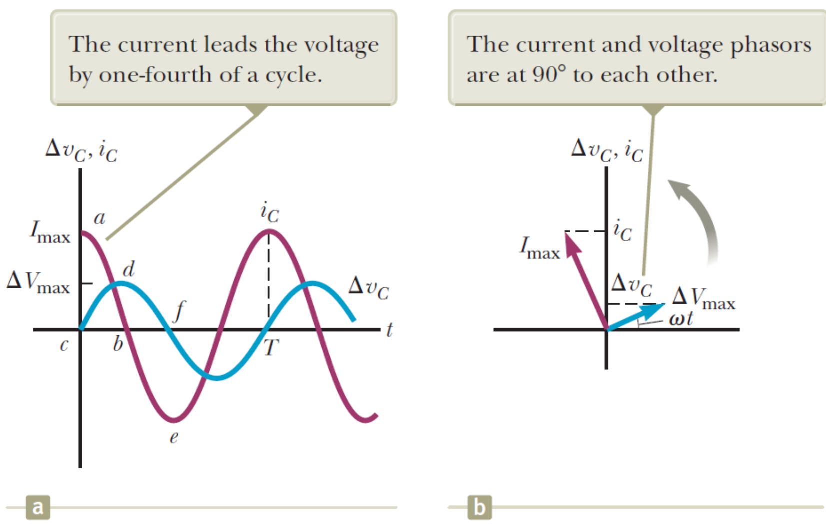
Using the trigonometric identity

$$\cos \omega t = \sin \left(\omega t + \frac{\pi}{2} \right)$$

we can express Equation 33.15 in the alternative form

$$i_C = \omega C \Delta V_{max} \sin \left(\omega t + \frac{\pi}{2} \right) \tag{33.16}$$

Comparing this expression with $\Delta v = \Delta V_{max} \sin \omega t$ shows that the current is $\pi/2$ rad = 90° out of phase with the voltage across the capacitor. A plot of current and voltage versus time (Fig. 33.10a) shows that the current reaches its maximum value one-quarter of a cycle sooner than the voltage reaches its maximum value.



<u>For an AC applied voltage, the current always leads the voltage across a capacitor by 90°.</u>

Equation 33.15 shows that the current in the circuit reaches its maximum value when $\cos \omega t = \pm 1$:

$$I_{\max} = \omega C \, \Delta V_{\max} = \frac{\Delta V_{\max}}{(1/\omega C)} \tag{33.17}$$

As in the case with inductors, this looks like Equation 27.7, so the denominator plays the role of resistance, with units of ohms. We give the combination $1/\omega C$ the symbol X_C, and because this function varies with frequency, we define it as the **capacitive reactance:**

$$X_C \equiv \frac{1}{\omega C} \tag{33.18}$$

We can now write Equation 33.17 as

$$I_{\max} = \frac{\Delta V_{\max}}{X_C} \tag{33.19}$$

The rms current is given by an expression similar to Equation 33.19, with $I_{\max}$ replaced by I_{rms} and $\Delta V_{\max}$ replaced by ΔV_{rms}.

Using Equation 33.19, we can express the instantaneous voltage across the capacitor as

$$\Delta v_C = \Delta V_{\max} \sin \omega t = I_{\max} X_C \sin \omega t \tag{33.20}$$